

Performance Analysis of Heave Motion of a Classic SPAR in the west Africa Region

¹Dokubo, I.T. ²Akandu, E.; and ³Dick, I.F

¹Department of Marine Engineering, Faculty of Engineering, Rivers State University, Port Harcourt, Rivers State, Nigeria.

^{1,2} Rivers State University, Port Harcourt, Nigeria.

Submitted: 10-05-2022

Revised: 19-05-2022

Accepted: 22-05-2022

ABSTRACT

Floating Spar platform has been proven to be an economical and efficient type of offshore oil and gas exploration structure in deep and ultra-deep water, however wave induced loads on Spar structures are very important factor that decides the performance rating of the response of the structure in question. These loads must be properly investigated during design or redesign stage to determine the response and structural integrity of the Spar Structure. This work is on the investigation of the heave responses of the spar structures due to induced wave loads at different angles of attack. To determine its most probable maximum heave amplitude at various angles of attack ranging from 0° to 90°. Froude-Krylov theory hypothesis was used to determine the heave force model. Spectra analysis in frequency domain was then carried out to obtain the required responses due to the force. A MATLAB-based program was developed to carry out these analyses from start to finish. This is to statistically determine the most probable maximum amplitudes of the spar Structure. The results obtained show that the highest maximum heave response amplitude is 1.235m at 90°.

Keywords: Spar Structure, Investigation, most probable maximum Heave amplitude, Wave induced loads.

I. INTRODUCTION

Spar can be supported by a drilling rig as well as top-tensioned production risers in water depths thousands of feet greater than the water depth limit for a tension leg platform (TLP). It is specially well equipped to support steel catenary risers (SCRs) using the pull-tube option, which allows the SCR to serve as a continuous welded steel containment for hydrocarbons from the seafloor to the topsides and protects the riser from vortex-induced vibration in the fastest part of the

wave profile. Another important innovation in the Spar design over other offshore structure is the mooring system. Since it has a minimal motion at the mooring connection points near the centre of Spar-pitch motions, the mooring legs may be designed as semi taut members with uplift on the anchors.

However, for the purpose of this paper we shall analyse the wave induced loads on a classic Spar structure and how difficult it is. This is based on the fact that waves in real sea conditions are very much irregular. Design engineers rely mostly on the formulae provided by standard classification societies to ensure accuracy, reliability and safety. However, it is advisable for them to have adequate knowledge and understanding on the theory formulation used in the statistical wave loads determination.

Response of the spar structure to induced wave loads are usually determined using various approaches. In this context, spectra analysis will be used to determine the maximum responses in heave motion.

Heave Motion

This is the linear motion along the vertical Z-axis and also the up and down motion of a ship as large swells.

Classic Spar in Brief

Classic spar basically comprises of a cylindrical hull with deep draft that provide excellent motion characteristics, the ambient deep current becomes the main problem. To solve this problem the truss spar concept was introduced. The upper part of the truss spar retains the cylindrical section of the classic spar, connected by the mid part truss system divided by heave plates and bottom part supported by soft tank fixed ballast. The third generation of spar system was introduced which is the cell spar. The cell spar is a

combination of smaller sized hulls surrounding the center cell that provides buoyancy. The diagram below shows the three types of spar currently in operations.



Fig. 1: Types of Spar (a) Classic Spar, (b) Truss Spar, (c) Cell Spar

Aim

The aim is to analyze the impact of induced wave load on the Spar, and to determine the most probable maximum amplitude for heave motions.

Objectives

- To develop mathematical models for heave motions using Froude-Krylov theory.
- To use Pierson-Moskowitz energy Spectrum to carry out the analysis
- To develop the MATLAB-based routine to calculate the various wave loads and, for simulating the dynamic behaviour of the Spar in West Africa Sea State.

Various methods have been adopted by different researchers to investigate ship responses, such as numerical method, using equation of motions, hybrid, and time history to carry out spectra analysis. All these gave good and satisfactory results after their estimations.

II. ANALYTICAL MODEL

In this research, an independent MATLAB-based program will be developed to carry out analysis for all the different angles of attacks on the spar structure. The Froude-Krylov force (Pressure force) developed and the Pierson-Moskowitz energy equation will be used to determine the spar structure most probable maximum heave amplitude by carrying out spectra analysis for angles $0^{\circ}:15^{\circ}:90^{\circ}$. After which the angle with the highest response is obtained, and the determined results validated with results extracted from work done in a different software to ascertain the accuracy of the method adopted in this research work.

III. METHODOLOGY

The materials used are the principal dimensions of the spar structure, the met ocean data for typical west Africa Sea state from DNVGL. The dimensions are as given below in a tabular form.

Table 1: SPAR Dimensions

SPAR DIMENSIONS	
Diameter	59.10 m
Length	16.46 m
Draft	4.0 m
Weight	1,7572 m
Water depth	1,642.7 ton

The met ocean data for the wave parameters are as given below for west Africa Sea state according to Thayamballi (2018).

Table 2: West Africa met ocean Data

Parameters	Coefficient value	
H_{max} (m)	T_z (s)	H_s (m)
Wave	5.1	7.6
		3.7
Swell	6.8	13.9

Definitions

H_s Significant wave height, H_{max} Maximum wave height, and T_z is the zero crossing period.

Determination of the Heave Force model

The wave load experience by offshore structural elements depends upon their geometry, (The size of these elements relative to the wavelength and their orientation to the wave propagation). The vertical wave load on a fixed cylinder is shown in the figure below.

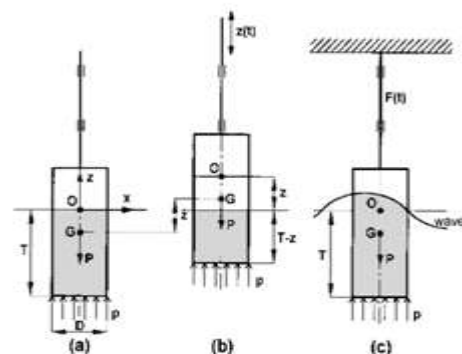


Fig.2: Heaving Circular Cylinder

The classic theory of deep-water waves yields

$$\text{Wave potential; } \varphi = \frac{\xi_a g}{w} e^{kz} \sin(\omega t - kx) \quad (1)$$

So that the pressure P, on the bottom of the cylinder (Z= -T) follows from linearized Bernoulli equations.

$$P = \rho \frac{d\varphi}{dt} - \rho g z = \rho g \xi_a e^{kz} \cos(\omega t - kx) - \rho g z \\ = \rho g \xi_a e^{-kT} \cos(\omega t - kx) + \rho g z \quad (2)$$

Assuming that the cylinder diameter is small relative to the wavelength (KD = 0), so that the pressure distribution on the bottom of the cylinder is essentially uniform, then the become (Gudnestad 2012)

$$P = \rho g \xi_a e^{-kT} \cos(\omega t) + \rho g T \quad (3)$$

Then the vertical force on the bottom of the cylinder is;

$$\frac{F}{\xi_a} = \{ \rho g e^{-kT} \cos(\omega t) + \rho g T \} * \frac{\pi}{4} D^2 \quad (4)$$

Where D is the cylinder diameter and T is the draft.

Similarly, from Velocity Potential φ

For deep water conditions:

$$\varphi = \frac{\xi_a g}{w} e^{kz} \cos(\omega t - kx) \quad (5)$$

The heave force or vertical force will be obtained by analyzing the pressure force (Froude Krylov)

$$\int dF_k = \int P_D dA \quad (6)$$

Where $P_D = -P_{dt}^{\varphi}$, $dA = 2\pi r dr$ Substituting into equation. 6

$$F_k = \int_0^{D/2} -\rho \frac{\partial \varphi}{\partial t} \cdot 2\pi r dr \quad (7)$$

$$\frac{\partial \varphi}{\partial x} = \xi_a k c e^{-kz} \sin(kx - \omega t) \quad (8)$$

Differentiating equation.8

$$\frac{\partial}{\partial t} \left(\frac{\partial \varphi}{\partial x} \right) = \xi_a k c w e^{-kz} \cos(kx - \omega t)$$

Substituting into equation 7

$$F_k = \int_0^{D/2} \rho \xi_a k c w e^{-kz} \cos(kx - \omega t) \cdot 2\pi r dr \quad (9)$$

Now integrating equation 9

$$F_k = 2\pi \rho \xi_a k c w e^{-kz} \cos(kx - \omega t) \int_0^{D/2} r dr \quad (10)$$

$$F_k = 2\pi \rho \xi_a k c w e^{-kz} \cos(kx - \omega t) \cdot \frac{D^2}{8} \quad (11)$$

$$F_k = \frac{\pi D^2}{4} \rho \xi_a k c w e^{-kz} \cos(kx - \omega t) \quad (12)$$

At the maximum amplitude x=0 and K=-D from the free surface

$$F_k = \frac{\pi D^2}{4} \rho \xi_a k c w e^{-kD} \cos(\omega t) \quad (13)$$

Hence the amplitude of the heave force will be expressed as per unit amplitude of the force

$$\frac{F_K}{\xi_a} = \frac{\pi D^2}{4} \rho k c w e^{-kD} \quad (14)$$

For deep water $c = \frac{g}{w}$

$$\frac{F_K}{\xi_a} = \frac{\pi D^2}{4} \rho k g e^{-kD} \quad (15)$$

Definition of terms

The quasi-static force will be used with the magnification factor to determine the response amplitude operator (RAO) of the structure.

It is given as;

$$\frac{F_q}{\xi_a} = \frac{F_a}{C} \quad (16)$$

It will be worthy to note that C is the restoring force,

The dynamic amplification or magnification factor for a non-damped system is gotten from the equation shown.

$$\varphi = \frac{1}{\sqrt{(1 - R^2)}} \quad (17)$$

R is the frequency ratio, and is expressed as;

$$R = \frac{\omega}{\omega_n} \quad (18)$$

The natural frequency of the structure is then obtained with the equation given below.

$$\omega_n = \sqrt{\frac{C}{M + A}} \quad (19)$$

The response amplitude operator (RAO) is used to measure the response of the structure exposed to first order wave loads.

$$RAO = F_q \cdot \varphi \quad (20)$$

The Pierson-Moskowitz wave energy spectrum equation is used with the square of the RAO to determine the response in various angles of attacks. And it is given as; determinethe response in various angles of attacks. And it is given as;

$$S(\omega) = \frac{124}{T_z^4} H_s^2 \omega^{-5} \exp\left(-\frac{496}{T_z^4} \omega^{-4}\right) \quad (21)$$

Hence, the ship responses will be achieved using the equation as given.

$$S_R = S(\omega) \cdot RAO^2 \quad (22)$$

The variance or Zeroth Moment (M_0) is achieved by using the equation below

$$M_0 = \frac{0.001 * S_{RT}}{3} \quad (23)$$

The significant Heave amplitude (ξ_h) is expressed as given below.

Where S_{RT} is the sum of the responses, and 0.001 is the step size chosen during the wave number determination which will be seen in the results and discussions section of this work.

$$\xi_h = 2 \cdot \sqrt{M_0} \quad (24)$$

Then the most probable maximum surge amplitude (ξ_{max}) will be obtained with the equation below.

$$\xi_{max} = 1.86 * \xi_h \quad (25)$$

IV. RESULT AND ANALYSIS

Range of wave number 0.001:0.001:3.001 rad.m²/s were used in equation (19) to determine the wave frequencies used for the analysis. This gave 3001 values of the wave frequencies; it implies that for each parameter analyzed, results obtained are within this range for more statistical accuracy. The maximum results for each angle of attack 0, 45, and 90 degrees are of focus. As shown in the graph below, the impact of the heave force will be maximum all through from 0 to about 4.8 rad/s and breaks as it tends towards 5rad/s. which simply implies the structure will experience maximum impart and response throughout the wave frequencies until it breaks off.

From the results above, the highest value of the maximum amplitude of the heave forces for all the angles occurred at 90⁰ angle of attack,

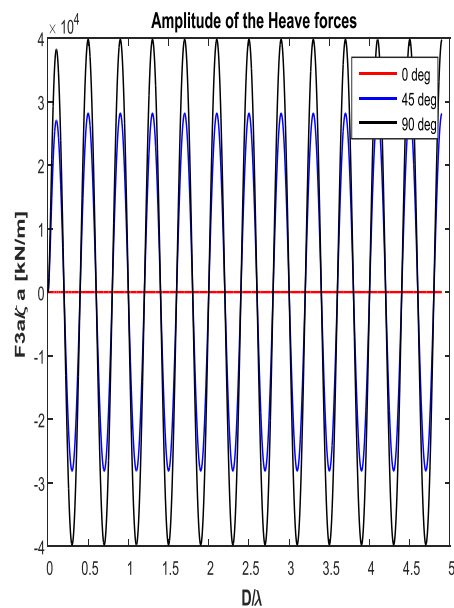


Fig. 3: Heave Force per wave amplitude (0⁰-90⁰)

A. Response Amplitude Operator (RAO)
 For the RAOs, the maximum for the heave motion at 90 deg with 1.1010. For 45 deg, heave motion is 0.7785 respectively. The Heave response amplitude operator is obtained by multiplying the Quasi-Static force and the magnification factor, the maximum response occurred at around 0.2 rad/s and dies off as it tends towards 3rad/s. as shown below

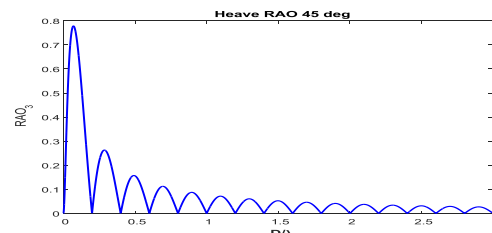


Fig. 4: Heave RAO 45 deg

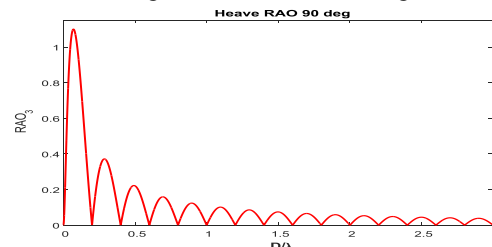


Fig. 5: Heave RAO 90 Deg

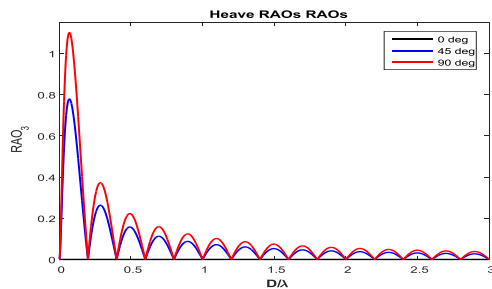


Fig. 6: Heave RAO 45 and 90 Deg

B. Spar Response Spectra

The response of the Spar is obtained by making use of equation (21) for all the various angles of attacks. Plots of the responses against the frequencies form the response spectrum of the Spar. The response is referred to as the area under the spectrum curve which will be seen in the figures below. The highest response for the heave angle is as at; $90^0 = 0.12\text{rad/s}$, with a numerical value $49.8579\text{m}^2/\text{s}^2/\text{rad}$. Figures below show the plot of all the responses from 0^0 to 90^0 , they showed clearly the behaviour and the pattern with which the Spar is responding to the induced wave.

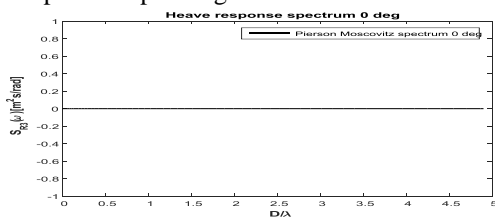


Fig. 7: Heave Response Spectrum 0 Deg

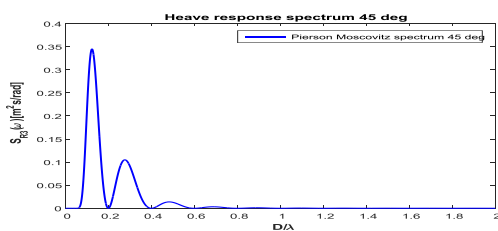


Fig. 8: Heave Response Spectrum 45 Deg

V. CONCLUSION

The most probable maximum heave amplitude represents the maximum responses of the spar in Heave motion as a result of the induced wave loads. From the statistical results in the Table 2 above, the 90^0 angles of attack gave the highest response of the Spar. Response in this context implies the vertical displacement from the mean perpendicular position. These responses are considered as the area under the spectra multiplied

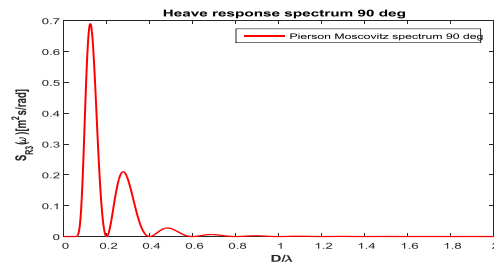


Fig.9: Heave Response Spectrum 90 Deg

As seen in the figures above, the graphs are similar as expected for all the angles of attacks.

C. Most Probable Maximum Heave Amplitude (ξ_{max})

This is the highest response of the spar statistically determined. For this to be obtained, the variance (Zeroth Moment) will be obtained using Simpson's first rule multiplier, after which the significant heave amplitude will also be gotten. Then the most probable maximum heave amplitude will then be achieved for all the various angles of attacks. using equations (23), (24) and (25) respectively.

The final results are tabulated for the various angles of attacks. These results represent the ship final responses. As said earlier, major interest is on the determination of the most probable maximum surge amplitude, which has achieved with all these procedures in Table 2 below.

Table 2: Spar Response Results 0-90⁰

Angle (deg)	M_{01} (m ²)	ξ_h (m)	ξ_{max} (m)
0	0	0	0
45	0.0552	0.4698	0.8738
90	0.1104	0.6644	1.2358

by Simpson's multipliers and the step size, as shown in figures 7 to 9 above.

The response amplitude operator (RAO) reviewed that the dynamic resonance of the Spar structure occurred around the first order wave frequencies. From the RAO graphs, resonance occurred where the natural frequency of the structure is equal to the wave frequency.

The deep-water dispersion characteristics equation used in a reversed manner (that is to estimate a range of wave numbers to be able to determine the wave frequencies) used in this work

has proven to be a good approach for deep water wave analysis.

The heaveforce model developed above using Froude-Krylov hypothesis proved to be a good method to evaluate the pressure force acting on the structure. After all the analyses and validation made in course of this research, the results have shown that the model developed is suitable for offshore structures of similar shape and characteristics. And also, the software developed is simple and easy to understand, and can be used for analysis of similar offshore structures.

Based on the results and findings in this research, the highest most probable maximum heave amplitude occurred at 90°

REFERENCES

- [1]. Akandu, E. (2017). "Dynamics of Offshore Structures". (Lecture Note), Rivers State University, Department of Marine Engineering, Port Harcourt, Nigeria.
- [2]. Akandu, E. (2018). "Marine Hydromechanics and Environment" . (Lecture Note), Rivers State University, Department of Marine Engineering, Port Harcourt, Nigeria.
- [3]. Buchner, B. (2006). "The motion of a ship on a sloped seabed. International conference on offshore mechanics and Arctic engineering; 25th, (pp.4-6) Hamburg Germany.
- [4]. Bayati, I. S. A. (2015). "Study of the effect of water depth on potential flow solution of the semisubmersible floating offshore wind turbine. Energy Procedia (pp. 168-176)
- [5]. Bander, S. I. (2013). "Dynamic Responses of offshore Spar Platform due to wave-current interaction.(Masters Thesis), Norwegian University of Science and Technology, Department of Marine.
- [6]. Chem, J. (2014). "Nonlinear Wave Loads on Offshore Wind Support Structure. (Master Thesis) Norwegian university of science and technology.
- [7]. Dokubo, I. T (2021). "Performance Analysis of a Classic SPAR in the West African Region. Master Dissertation, Rivers State university, Department of Marine Engineering, Port Harcourt.
- [8]. DNV-OS-J101.(2010). "Design of Offshore wind Turbine structure Standard.
- [9]. Faltinsen, O. M, Newman, and Vinje, T.(1995). "Nonlinear wave loads on a slender vertical cylinder. Journal of fluid mechanics, 289: 175-198.
- [10]. Gudmestad, O. "Marine technology & Design University of Stavanger, Department of Marine Technology Norway.
- [11]. Journée, W. (2001). Offshore Hydrodynamics. Delft University of technology. First Edition, (Lecture Note OT3600). Retrieved June 11, 2016, from Delft university. <https://www.shipmotions.nl>
- [12]. Klepšvik, J. (1995). Nonlinear wave loads on offshore structures. Research Gae Conference Paper, (pp. 1-2). Norway